On the lump instability of the Davey-Stewartson - II equation.

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The Davey-Stewartson equation (DS-II) is the well-known example of the 2+1 dimensional integrable equation:

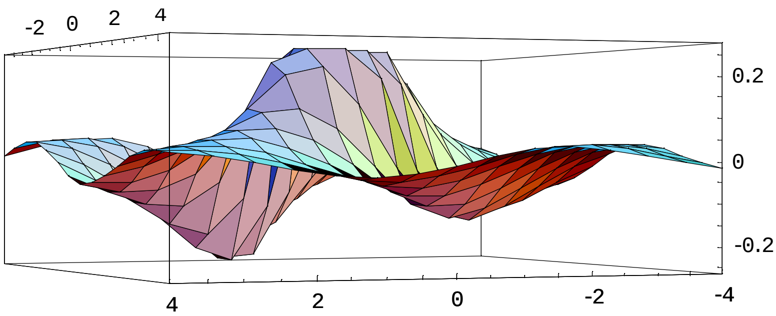
$$i¶_t q + 2(¶_z^2 + ¶_{\bar{z}}^2)q + (g + \bar{g})q = 0,$$

$$¶_{\bar{z}} g = ¶_z |q|^2, \qquad z \in \tilde{N}.$$

This equation has the soliton solution (Arkadiev, Pogrebkov, Polivanov,1989):

$$q_0(z,t) = \frac{2\bar{n}\exp(k_0 - \overline{k_0 z} + 2it(k_0^2 + \bar{k}_0^2))}{|z + 4ik_0 t + m|^2 + |n|^2},$$

$$g(z,t) = \frac{-4\overline{(z + 4ik_0 t + m)^2}}{(\{z + 4ik_0 t + m|^2 + |n|^2)^2}.$$



We shall study the Cauchy problem with initial data in the form of the perturbed soliton:

$$q_e(z,0) = q_0(z,0) + e q_1(z),$$

where $q_1(z)$ is smooth function with a finite support and ♏ is small positive parameter.

The main question: What's happened with the perturbed soliton initial data?

The perturbation of the scattering data

The scattering problem for the Dirac equation is associated with the DS-II equation:

$$\begin{pmatrix} ¶_{\bar{z}} & 0 \\ 0 & ¶_z \end{pmatrix} f = \begin{pmatrix} 0 & q/2 \\ -\bar{q}/2 & 0 \end{pmatrix} f, \qquad \begin{pmatrix} \exp(-kz) & 0 \\ 0 & \overline{\exp(-kz)} \end{pmatrix} f \to \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |z| \to \infty.$$

The scattering data for the problem has two different parts:
the discrete part {k,❍✍■“   and the continuous part:

$$b(k,t) = \frac{1}{4p} \iint dz \wedge d\bar{z}\, \overline{q(z,t) f_1} \exp(-\overline{kz}).$$

For the pure soliton solution the scattering data has the form:

$$\{k_0, m_0, h_0\}; \qquad b(k) \equiv 0.$$

In the soliton case the solution of the boundary problem for the Dirac system has a pole with respect to k in the point $k=k_0$, and the system has a decreasing at $|z| \to \infty$ nontrivial solution.

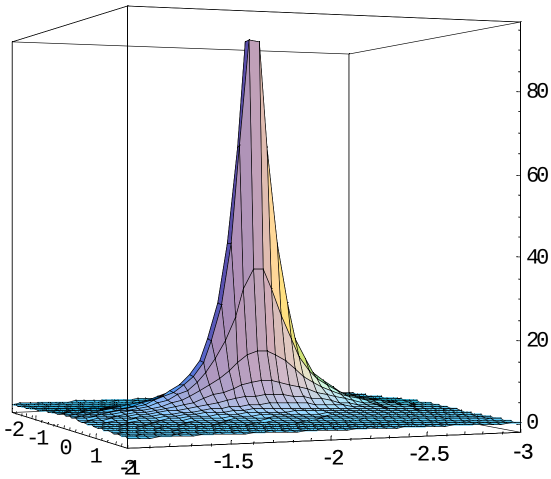
Study the scattering data under perturbation . As the result we obtain (Gadyl’shin, Kiselev 1996):

$$b_e(k) = e b_1(k) + o(e), \quad |k-k_0| > C e^g, \quad 0 \le g < 1,\ C > 0;$$

$$b_e(k) = e^{-1} B_{-1}\left(\frac{k-k_0}{e}\right) + O(1), \quad |k-k_0| = O(e^d), \quad 0 < d \le 1.$$

where $B_{-1}(z) = \dfrac{4ipQ_1}{|Q_1|^2 + |Q_2 + 4ipz|^2}$, $Q_{1,2}$ is some constants depending on the perturbation.

<u>Conclusion 1. The scattering data has nonsoliton structure under perturbation of the pure soliton initial data.</u>



The example. $q_1 = iq_0(z)$. Perturbation parameter e=0.1, Q_1=1. There is the continuous part of the scattering data.

The question is: Why does the discreet part of the scattering data disappear?

The eigenvalue problem

Consider the integral equation which is equivalents to the boundary problem for the Dirac equation.

$$(I - G[q,k])f = E(kz), \quad where \quad E(kz) = diag(\exp(kz), \exp(\overline{kz})).$$

The solution of this equation has the pole if the eigenvalue problem

$$(I - G[q,k])f = l f$$

has the nil of the eigenvalue: l=0. Then the soliton and the nil of the eigenvalue are associated with each other.

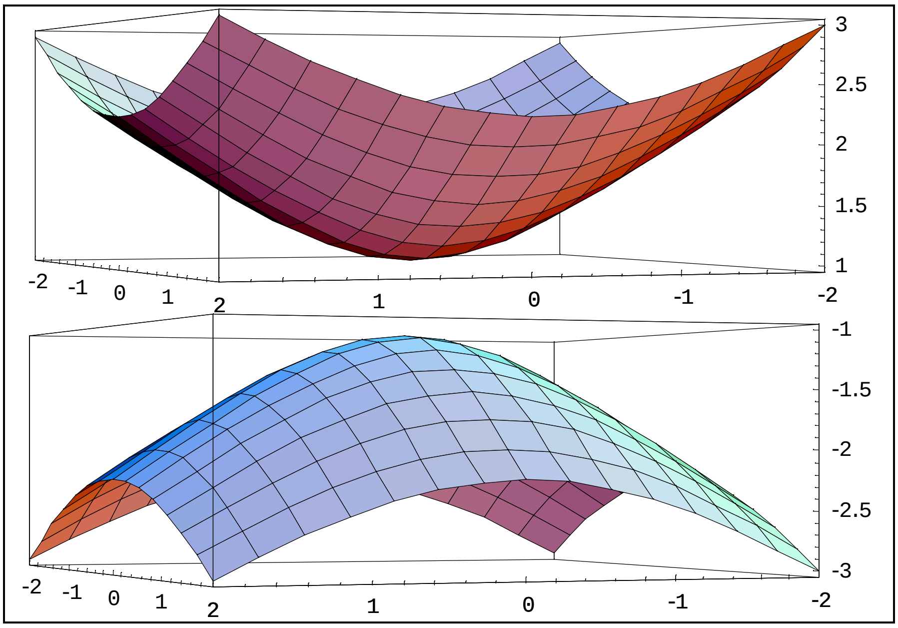
We studied this eigenvalue problem under perturbation of the q(z) near the k=k_0 locally. As the result we obtained (Gadyl'shin, Kiselev 1998), that the eigenvalue which is associated to the pure soliton function is semisimple. Under the perturbation it decomposes to two simple eigenvalues:

$$l_{1,2} = \pm e \overset{1}{l}(k) + o(e).$$

Locally we obtained that the $\overset{1}{l}(k)$ is the solution of the equation:

$$(\overset{1}{l} + Q_1)(\overset{1}{l} + \overline{Q}_1) + |\overline{Q}_2 - z|^2 = 0, \quad where \quad z = \frac{k - k_0}{e}.$$

<u>Conclusion 2. The eigenvalue is not equal nil if Q_1 not equal to zero.</u>



The example of the perturbation for the semisimple eigen-value. (Q_1 =1, The scale order is ɱ$^{-1}$. The parameter k-k_o is lies on the horizontal plane. The figure is shows the eigen-values as a function with respect to k- k_o .

The question is: What's happened with the solution of the DS-II equation?

<u>The asymptotic solution as t=O(ɱ$^{-1}$)</u>

We obtained the asymptotic solution of the Cauchy problem for the DS-II equation by the inverse scattering method. For this we solved the D-bar problem:

$$\begin{pmatrix} \partial_{\bar k} & 0 \\ 0 & \partial_k \end{pmatrix} \psi = \begin{pmatrix} 0 & b_\varepsilon \exp(iS) \\ \bar b_\varepsilon \exp(-iS) & 0 \end{pmatrix} \psi , \quad \psi \begin{pmatrix} \exp(-kz) & 0 \\ 0 & \exp(-\overline{kz}) \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad |k| \to \infty.$$

where $S = -i(kz - \overline{kz}) + 2t(k^2 + \bar k^2)$.

The solution for the DS-II equation has the following form:

$$q(z,t) = \frac{1}{\pi} \iint dp \wedge d\bar p \, b_\varepsilon(k) \psi_{11}(k,z,t) \exp(iS).$$

Using the form of the $b_\varepsilon(k)$ we obtain (Gadyl'shin, Kiselev 1997):

$$q_\varepsilon(z,t) = \frac{2\bar\nu \exp(k_0 - \overline{k_0 z} + 2it(k_0^2 + \bar k_0^2))}{|z + 4ik_0 t + \mu_\varepsilon|^2 + |\nu|^2} + o(\varepsilon t),$$

$$g_\varepsilon(z,t) = \frac{-4\overline{(z + 4ik_0 t + \mu_\varepsilon)^2}}{(|z + 4ik_0 t + \mu_\varepsilon|^2 + |\nu|^2)^2} + o(\varepsilon t).$$

Here $\mu_\varepsilon = \mu_0 - 2i\pi\varepsilon t Q_2$. It is the influence of the perturbation.

Conclusion 3. The perturbation of the initial data changes the soliton parameter μ only.The soliton-like solution propagates without the change of its shape at t=O(ε^{-1}).

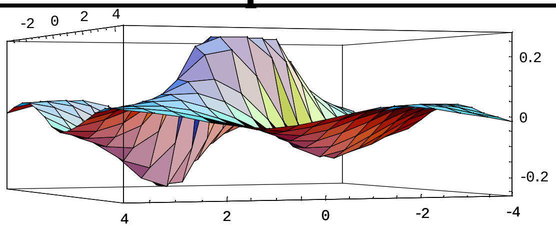
The question is: What is meant the solitonless scattering data?

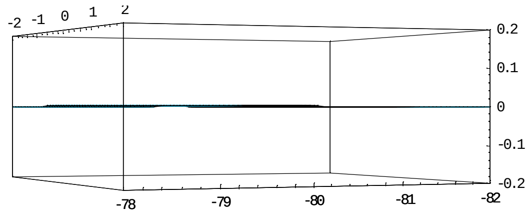
The long time asymptotics.

The nonsoliton solution for the DS-II equation on a long time has the form (Kiselev, 1996):

$$q = t^{-1}\frac{1}{2}b\left(\frac{i}{4t}(x+iy)\right)\exp\left(\frac{i}{4t}(x^2-y^2)\right)+O(t^{-5/4}),\quad as\quad t\to\infty.$$

Conclusion 4. The perturbed soliton solution disappear for t>> e^{-1} .





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